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The Impact of Exogenous Variables on Stock Market Volatility: A GARCH-X Model Approach

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Abstract: This study was conducted to investigate the impact of exogenous variables on stock price volatility. The performance of the classical GARCH model and GARCH-X models was compared using stock price data. The GARCH-X model incorporates two exogenous variables (exchange rate and interest rate), which allows it to account for external economic factors and asymmetries in the data. The study utilised daily stock prices, exchange rates, and interest rates from February 1, 2012, to February 5, 2024. The descriptive statistics revealed that the distributions of returns on the stock prices were skewed and leptokurtic. The result of the unit root test, conducted using the augmented Dickey-Fuller (ADF) test, indicated that the returns on the series were stationary. The ARCH LM-test detected the presence of ARCH effects, justifying the use of GARCH models. The mean equation was estimated, and the GARCH-X (1,1) model was fitted to the data, incorporating two exogenous variables (daily exchange rate and interest rate). The Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Hannan-Quinn (HQ), Shibata, and Log-Likelihood were used to test the models' goodness of fit. The study's findings revealed that GARCH-X (1,1) has the lowest AIC, BIC, and HQ compared to GARCH (1,1). Both models exhibit high persistence in volatility, with GARCH (1,1) at 0.98805 and GARCH-X (1,1) at 0.9989987, indicating that volatility shocks have prolonged effects. Furthermore, among the exogenous variables, $\delta 1$ was statistically significant at the 5% level ($p = 0.0147$), suggesting that the exchange rate has a significant effect on stock price volatility. However, $\delta 2$ was not significant ($p = 0.0842$), indicating that the interest rate does not have a significant impact on volatility within the model.

Keywords: Exogenous Variables; GARCH-X Model; Exchange Rate; GARCH and Volatility; Nigeria's Economy; Stock Market; Economic Stability; Macroeconomic Variables; Stock Volatility.

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1. Introduction

In developing economies like Nigeria, where various internal and external economic factors influence financial markets, understanding the factors driving stock volatility is critical for both foreign and local investors. Stock market volatility reflects

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the risks and uncertainties associated with asset prices, influencing investment decisions, portfolio management, and the overall stability of the economy. This study examines two fundamental macroeconomic variables—interest rates and exchange rates—and their impact on stock market volatility in Nigeria, providing insight into the complex dynamics of the country's financial markets. Interest rates are the cost of borrowing and serve as a primary tool for the Central Bank of Nigeria (CBN) to influence economic activity [9]. Interest rate fluctuations can impact a company's profitability, capital costs, and, ultimately, its stock prices, as investors adjust their expectations based on projected returns relative to borrowing costs.

In a developing economy like Nigeria, where monetary policy shifts are often driven by inflationary pressures and the need to maintain economic stability, adjustments in interest rates are expected to have significant effects on market volatility [13]. Exchange rates, on the other hand, could be described as the value of the Nigerian naira relative to other foreign currencies, especially the US dollar. Being one of the world's largest oil exporters, Nigeria's economy is significantly influenced by global petroleum prices [15]. Consequently, changes in oil prices induce volatility in foreign currency values, which in turn affects domestic financial circumstances and investor sentiment. Firms involved in international trade are particularly concerned with exchange rate fluctuations, as these changes can affect their earnings and investment outcomes [5]. Exchange rate fluctuations pose a risk to investors, as they have a direct impact on the value of foreign investments in local currency terms, thereby affecting stock prices and market stability.

The Nigerian stock market has experienced substantial volatility, driven by both domestic and international economic factors. For instance, when the currency depreciates, investors often reallocate their portfolios towards safer assets or international markets, thereby increasing volatility in the domestic market [7]. Similarly, rising interest rates can lead to a decline in stock prices, as investors shift their focus to higher-yield fixed-income assets, resulting in capital allocation adjustments and increased volatility in the stock market. Few studies have explicitly examined developing countries like Nigeria, despite prior research investigating how exchange rates impact stock markets in industrialised economies.

This study addresses a critical gap by examining the impact of changes in interest rates and exchange rates on stock volatility in Nigeria. It utilises the GARCH-X model, which enables the inclusion of exogenous variables to reflect better the complex relationship between macroeconomic factors and stock market volatility. Understanding how interest rates and exchange rates affect stock volatility in Nigeria can help investors, governments, and financial analysts reduce risks and develop stock market stabilisation techniques. The results of this study can help inform our approach to monetary policy and provide a foundation for more robust investment strategies in Nigeria's emerging market environment. This study employed the GARCH-X model to assess the influence of interest rates and exchange rates on stock market volatility in Nigeria. Thus, by providing empirical evidence on the relationship between macroeconomic variables and stock volatility for developing nations, we contribute to the existing literature by filling a gap in research on developing nations, particularly Nigeria, where stock markets may respond differently to macroeconomic fluctuations than those in developed markets.

2. Literature Review

The link between macroeconomic variables, such as interest rates, exchange rates, and stock market volatility, has received considerable attention in both developed and developing nations. Stock market volatility is a significant topic of research, as it reflects the level of risk and uncertainty in financial markets. According to Schwert [4], several factors can influence volatility, including economic data, market sentiment, and external shocks. Macroeconomic variables, notably interest rates and exchange rates, are significant drivers of stock price fluctuations and volatility. Numerous studies have found a strong correlation between interest rates and stock market performance. For instance, Chen et al. [8] found that changes in interest rates can affect stock prices through their influence on corporate profits and investment decisions.

An increase in interest rates typically leads to higher borrowing costs for firms, which may reduce profitability and consequently depress stock prices [3]. Interest rates are often closely tied to market volatility, as they directly influence the cost of capital and economic growth projections. Li and Peng [16] employed a GARCH-X model to examine the effect of interest rate changes on the Chinese stock market, finding that surprise rate cuts reduce volatility, suggesting that monetary policy signals alleviate investor anxiety. In the Nigerian context, research by Olofin and Afangideh [13] reveals that interest rate fluctuations have a significant impact on stock market volatility. Their study revealed that increases in interest rates tend to lead to heightened volatility, as investors adjust their portfolios in response to changes in the cost of capital. Similarly, Nwosa and Saibu [9] highlighted that tightening monetary policy, through increased interest rates, results in increased uncertainty among investors, contributing to greater volatility in the Nigerian stock market.

Exchange rates have also been shown to significantly affect stock market volatility, especially in economies that are heavily reliant on international trade and investment. A study by Goh et al. [2] demonstrated that exchange rate fluctuations could lead to changes in investor sentiment and expectations, influencing stock prices and market stability. In Nigeria, where the economy is heavily dependent on oil exports, exchange rate volatility poses significant risks to the stock market. Research by Oloko [15] found that exchange rate depreciation leads to increased stock market volatility, as investors react to the potential impacts on corporate earnings and overall economic stability. The study found that the sensitivity of stock prices to exchange rate movements is pronounced during periods of economic uncertainty, which is typical in developing nations like Nigeria.

GARCH models have become a popular method for modelling and forecasting volatility in financial markets. These models effectively capture the time-varying nature of volatility and have been widely applied in studies examining the relationship between macroeconomic variables and stock market behaviour. Engle [10] introduced the ARCH model, which was later generalised to GARCH by Bollerslev [14], allowing for more flexibility in capturing volatility dynamics. Studies applying GARCH models in emerging markets have yielded significant insights into the relationships among macroeconomic variables. For instance, a study by Chittedi et al. [6] employed GARCH models to investigate the impact of interest rates and exchange rates on stock market volatility in India, revealing that both factors significantly influenced volatility patterns. Similarly, Oduro and Owusu [1] employed a GARCH-X model to examine the relationship between oil prices, exchange rates, and stock market volatility in Ghana, highlighting the importance of incorporating exogenous variables into the analysis.

Exchange rate fluctuations are particularly relevant in open economies that are largely reliant on foreign trade. Hassan et al. [12] employed a GARCH-X model to examine the relationship between exchange rates and stock market volatility within the European Union. They discovered that exchange rate depreciation frequently promotes increased volatility by raising uncertainties about future financial success for businesses with overseas exposure. Despite the growing body of literature on stock market volatility and its determinants, there remains a limited focus on the Nigerian stock market, particularly regarding the joint impact of interest rates and exchange rates. While previous studies have explored individual effects, there is a need for comprehensive analyses that incorporate both variables simultaneously, particularly using advanced models like GARCH-X to capture the complexities of these relationships. This literature review highlights the established links between interest rates, exchange rates, and stock market volatility, with significant findings from both global and Nigerian contexts. By utilising a GARCH-X model, this study aims to contribute to the existing literature by providing empirical evidence on the dynamic interactions among these macroeconomic factors in Nigeria, addressing the identified gaps in previous research.

3. Methodology

3.1. Autoregressive Conditional Heteroskedasticity (ARCH) Models

Engle [11] proposed the ARCH model (Auto-regressive Conditional Heteroskedastic Model). Every ARCH or GARCH family model requires two distinct specifications: the mean equation and the variance equation. According to Chen et al. [8], conditional heteroskedasticity in a return series can be modelled using the ARCH model, expressing the mean equation in the form:

$$y_t = E_{t-1}(y_t) + \varepsilon_t \quad (1)$$

Where;

$\varepsilon_t = \varphi_t \sigma_t$
ε_t is the error gotten from the mean equation at time t
E_{t-1} is the expectation conditioned on the information available at time t-1
φ_t is a sequence of independent, identically distributed random variables whose mean is zero with unit variance
The variance equation for an ARCH model of order q is given as:

$$\sigma_t{}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^{\,2} + \mu_t$$

$$= \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\,2} + +\alpha_2 \varepsilon_{t-2}^{\,2} + \alpha_3 \varepsilon_{t-3}^{\,2} + \cdots + \alpha_p \varepsilon_{t-p}^{\,2} \quad (2)$$

Where

$\alpha_0 > 0$, $\alpha_i \geq 0$, $\varepsilon_{\,t-1}, \varepsilon_{\,t-2}, \ldots \varepsilon_{\,t-p}$ Are independent for all t and $i = 1,2,3, \ldots, q$ is the number of lags.

In the practical application of the ARCH (q) model, the decay rate is usually more rapid than what applies to financial time series data. To account for this, the order of the ARCH must be maximised, a process that is strenuous and more cumbersome. Thus, the GARCH model was introduced.

3.2. GARCH (p; q) Model

Although the ARCH model is simple, it often requires numerous parameters. To circumvent this difficulty, Bollerslev [14] proposed a useful extension of the ARCH model known as the generalised ARCH (GARCH) model. Their results suggested that the GARCH model better captures the volatility in a return series than the ARCH model. A stochastic process $\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_t$ is said to follow a GARCH (p; q) model if,

$$\sigma_t{}^2 = \beta_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon^{\,2}_{t-i} + \sum_{i=1}^{p} \beta_j \, \varepsilon^{\,2}_{t-j} \qquad (3)$$

Where:

p = order of the GARCH terms,
q = order of the ARCH terms, ε^2.
$\sigma_t{}^2 =$ error variance at time t.

$\beta_0 > 0, \alpha_i > 0, i = 1,2,3 \dots q-1, \;\; j = 1,2,3, \dots, p-1$

$\varepsilon^2{}_t$, disturbance term.

A stochastic process $\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_t$ is said to follow the GARCH (1,1) model if $\varepsilon_t = \sigma_t Z_t$ and

$$\sigma^2_t = \beta_0 + \beta_1 \varepsilon^2_{t-1} + \beta_2 \sigma^2_{t-1} \qquad (4)$$

Then the residual return or shock at time t is defined as:

$$R = \mu + \varepsilon_t \qquad (5)$$

Where,

$\varepsilon^2_t \sim \mu(0, \sigma^2_t)$
μ is the mean of the return
σ^2_t is the error variance at time t
ε^2_{t-1} is the squared error at time t-1

The three parameters (β_0, $\beta_1$andβ_2) are nonnegative and β_1+β_2<1 to achieve stationarity.

3.3. GARCH-X Model (Generalised Autoregressive Conditional Heteroskedasticity with Exogenous Variables)

The GARCH-X model is an extension of the traditional GARCH model, incorporating exogenous variables to account for additional factors influencing volatility. The model's mean equation could be specified as follows:

$$r_t = \mu + \sum_{i=1}^{n} \delta_i \, X_{i,t} + \varepsilon_t \qquad (6)$$

Where:

r_t Is the return at time t, μ is the constant mean return, $X_{i,t}$ is the exogenous variable with a coefficient $\delta_i, i = 1,2,3 \dots, n$ and ε_t The error that is often assumed to follow the skewed student's t-distribution.

The conditional variance equation is given as follows:

$$\sigma^2_t = \omega + \sum_{j=1}^{p} \alpha_j \, \varepsilon^2_{t-j} + \sum_{i=1}^{q} \beta_i \, \sigma^2_{t-i} + \sum_{m=1}^{n} \gamma_m \, Z_{m,t} \qquad (7)$$

Where:

σ_t^2 is the conditional variance at time t, ω is the constant variance term α_j Is the coefficient of the ARCH term, which captures the influence of past error terms up to lag p, and $Z_{m,t}$ Is the exogenous variables that affect the variance, with a coefficient γ_m.

3.4. Error Distributions

Stock return probability distributions often have larger tails than the standard normal distribution. Heavy-tailedness is most likely caused by volatility clustering in stock markets. Furthermore, sudden variations in stock returns appear to be another source of heavy-tailedness. Fat tails may also contribute to increased kurtosis. In practice, the outcomes are frequently skewed. To completely depict this phenomenon, this study also considered the skewed Student-t distributions. In the case of the t-innovation, the volatility models are estimated to maximise the probability function of the distribution as follows:

$$L_{(std)}(\theta_t) = -\frac{1}{2}\ln\left(\frac{\pi(d)\Gamma\left({}^{d}/_{2^2}\right)}{\Gamma\left[\left({}^{d+1}/_{2}\right)\right]^2}\right) - \frac{1}{2}\ln S_t^2 - \frac{d+1}{2}\left(1 + \frac{\left(r_t - X_{t,\theta}\right)^2}{S_t^2\,(d-2)}\right) \quad (8)$$

In the case of the skewed student's t-innovation, the volatility model is to maximise the likelihood function of the skewed student's t distribution. The probability density function of the skewed student's t distribution is:

$$f(x) = \begin{cases} \frac{bc}{w}\left(1 + \frac{1}{v}\left(\frac{b(x-\varepsilon)}{w}\right)^2\right)^{-\frac{v+1}{2}}, if\ x\ \geq \varepsilon \\ \frac{bc}{w}\left(1 + \frac{1}{v}\left(\frac{b(x-\varepsilon)}{w}\right)^2\right)^{-\frac{v+1}{2}}, if\ x\ < \varepsilon \end{cases} \quad (9)$$

The log of the likelihood function is given as follows:

For x ≥ 0:

$$logL(x;\ v,\lambda) = log\left(\frac{2}{v+1}t\left(\frac{x}{v}\right)T\left(\lambda.\frac{x}{v+1}, v+1\right)\right)$$

$$= log\left(\frac{2}{v+1}\right) + log\left(t\left(\frac{x}{v}\right)\right) + log\left(T\left(\lambda.\frac{x}{v+1}, v+1\right)\right) \quad (10)$$

Log-likelihood for x < 0:

$$logL(x;\ v,\lambda) = log\left(\frac{2}{v+1}.t\left(\frac{-x}{v}\right)[T-1\left(\lambda.\frac{x}{v+1}, v+1\right)]\right)$$

$$= log\left(\frac{2}{v+1}\right) + \log\left(t\left(\frac{-x}{v}\right)\right) + log[T-1\left(\lambda.\frac{x}{v+1}, v+1\right)]\Big) \quad (11)$$

3.5. Model Selection

Model selection is performed using information criteria, and the model with the lowest information criteria value across the error distributions is deemed the best-fitted. If the number of parameters in the model is denoted as p, then the AIC is defined by:

$$AIC(p) = -2In(Ml) + 2p \quad (12)$$

Where: Ml is the maximum likelihood estimate.

The Bayesian information criteria (BIC) given by

$$BIC(p) = -2In(Ml) + pIn(N) \quad (13)$$

Where:

N is the number of observations. For a large dataset, the BIC imposes a heavier penalty for the number of parameters in the model; therefore, it selects a more parsimonious model. This selection yields the best-fitting conditional variance models for stock price returns, as shown in Table 1.

3.5.1. Data Collection

The study employed secondary data covering daily prices from January 2, 2012, to February 5, 2024, sourced from [17].

4. Results and Discussion

The descriptive statistics of Nigeria's daily stock prices, exchange rates, and interest rates reveal distinct trends. The stock price has a high average value (mean: 1667.151) and is highly volatile, as seen by its standard deviation of 631.7519 and wide range of 871.26 to 3984.18. Its distribution is positively skewed (skewness: 1.977883) and exhibits high kurtosis (7.070299), indicating the presence of frequent outliers. The Jarque-Bera test result of 4234.986 reveals that the stock price data does not follow a normal distribution. In contrast, the exchange rate and interest rate returns exhibit much lower variability, with smaller ranges (exchange rate return: 142.5601 to 156.8649, interest rate: 0.084416 to 0.119008) and standard deviation, suggesting relative stability in these markets.

Table 1: Descriptive statistics of the daily return series of Nigeria stock prices, exchange rate, and interest rate

	Stock Price	Exchanger Rate	Interest Rate
Mean	1667.151	150.1619	0.099844
Median	1585.560	150.1393	0.099808
Maximum	3984.180	156.8649	0.119008
Minimum	871.2600	142.5601	0.084416
Std. Dev.	631.7519	7.5163	0.004865
Skewness	1.977883	0.4127	0.044718
Kurtosis	7.070299	5.953334	4.959410
Jarque-Bera			
Probability	4234.986	0.286491	1.268096

Their distributions are asymmetric, as shown by their low skewness (exchange rate: 0.4127, interest rate: 0.044718) and kurtosis values close to three (exchange rate: 2.953334, interest rate: 2.959410) (Figure 1).

Figure 1: Time plot of the daily stock price series from February 2, 2012, to February 5, 2024

Additionally, the probability values for the Jarque-Bera test (interest rate: 1.268096) suggest that daily fluctuations in Nigeria's exchange rate and interest rate follow a more predictable pattern than those of stock prices. Overall, while the stock market shows signs of higher risk and unpredictability, the exchange rate and interest rate appear stable over the observed period. The Augmented Dickey-Fuller (ADF) test results for daily stock prices, exchange rates, and interest rates reveal that the test statistics for each variable are significantly lower than the critical values at all conventional levels (-3.43, -2.86, and 2.57). The p-values are 0.0000 for each variable, indicating strong evidence against the null hypothesis of a unit root. Thus, we conclude that the returns on daily stock prices, exchange rates, and interest rates are stationary. Table 3 presents the results of the ARCH-LM test, which examines the null hypothesis of no ARCH effects. The test statistic is a Chi-squared value of 370.93 with 12 degrees of freedom. The p-value is extremely low at 2.2 × 10^ (-16), indicating strong evidence against the null hypothesis. This suggests the presence of ARCH effects in the data. With these outcomes, fundamental conditions for applying GARCH family models have been fulfilled (Table 2).

Table 2: Unit root test for the return on the daily stock price

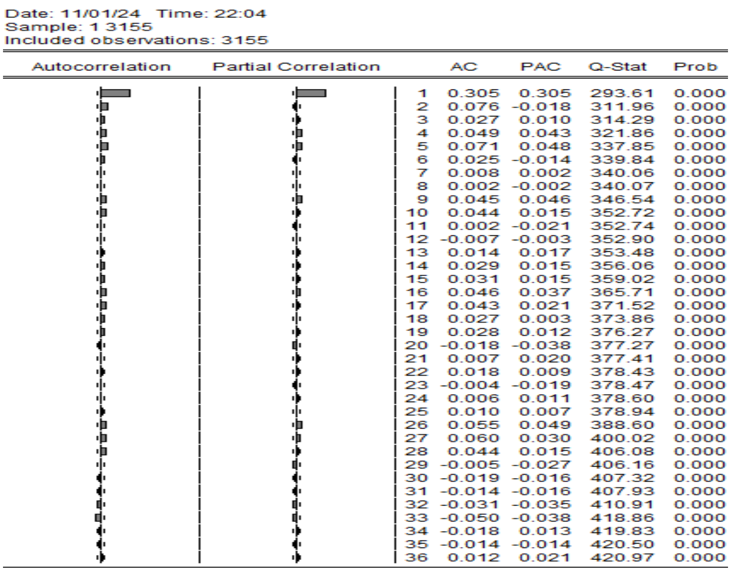
Null Hypothesis: PRICE has a unit root					
			Stock Price	**Exchanger Rate**	**Interest Rate**
Augmented Dickey-Fuller test statistic			-40.97818	-21.39410	-21.39410
Test critical values	-3.432234	-3.432248	-3.432248	-3.432246	-3.432246
	-2.862258	-2.862264	-2.862264	-2.862263	-2.862263
	-2.567196	-2.567200	-2.567200	-2.567199	-2.567199
	p-value		0.0000	0.0000	0.0000

Table 4 summarises the parameter estimates for a GARCH-X (1,1) model, incorporating three exogenous variables and a skewed Student's t-distribution for innovation. Key parameters include the mean (μ\muμ), two exogenous variable coefficients (δ1\delta_1δ1and δ2\delta_2δ2), variance constant (ω\omegaω), and the GARCH terms (α1\alpha_1α1 and β1\beta_1β1). Notably, α1, α1, and β1 have significant t-values and low p-values (below 0.05), indicating they are statistically significant contributors to volatility modelling. The high t-value and zero p-value of β1 highlight its strong influence. The skew parameter is also statistically significant, showing asymmetry in the data, which the skewed student's t-innovation accounts.

Table 3: ARCH LM-test: null hypothesis: no ARCH effects

Chi-squared	370.93
Df	12
p-value	2.2e-16

Model persistence is close to 1 (0.9989987), indicating high volatility persistence, where shocks to volatility decay very slowly, as reflected by the large half-life of 691.88 periods. This persistence suggests that the model anticipates volatility to persist at elevated levels for an extended period following a shock.


Date: 11/01/24 Time: 22:04
Sample: 1 3155
Included observations: 3155

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.305	0.305	293.61	0.000
		2	0.076	-0.018	311.96	0.000
		3	0.027	0.010	314.29	0.000
		4	0.049	0.043	321.86	0.000
		5	0.071	0.048	337.85	0.000
		6	0.025	-0.014	339.84	0.000
		7	0.008	0.002	340.06	0.000
		8	0.002	-0.002	340.07	0.000
		9	0.045	0.046	346.54	0.000
		10	0.044	0.015	352.72	0.000
		11	0.002	-0.021	352.74	0.000
		12	-0.007	-0.003	352.90	0.000
		13	0.014	0.017	353.48	0.000
		14	0.029	0.015	356.06	0.000
		15	0.031	0.015	359.02	0.000
		16	0.046	0.037	365.71	0.000
		17	0.043	0.021	371.52	0.000
		18	0.027	0.003	373.86	0.000
		19	0.028	0.012	376.27	0.000
		20	-0.018	-0.038	377.27	0.000
		21	0.007	0.020	377.41	0.000
		22	0.018	0.009	378.43	0.000
		23	-0.004	-0.019	378.47	0.000
		24	0.006	0.011	378.60	0.000
		25	0.010	0.007	378.94	0.000
		26	0.055	0.049	388.60	0.000
		27	0.060	0.030	400.02	0.000
		28	0.044	0.015	406.08	0.000
		29	-0.005	-0.027	406.16	0.000
		30	-0.019	-0.016	407.32	0.000
		31	-0.014	-0.016	407.93	0.000
		32	-0.031	-0.035	410.91	0.000
		33	-0.050	-0.038	418.86	0.000
		34	-0.018	0.013	419.83	0.000
		35	-0.014	-0.014	420.50	0.000
		36	0.012	0.021	420.97	0.000

Figure 2: Autocorrelation and partial autocorrelation plots with q-statistics and p-values

The overall structure, with significant exogenous terms and a high persistence measure, suggests that the model effectively captures the volatility dynamics influenced by both external factors and past variance. At the same time, skewness addresses asymmetry in returns (Figure 2). Table 4 presents estimates for the impact of three exogenous variables (represented by δ1, δ2, and the implicit intercept μ) in the GARCH-X (1,1) model. The coefficient δ1=0.007095\delta_1 = 0.007095δ1=0.007095 has a relatively low standard error and a p-value of 0.0147, indicating that it is statistically significant at the 5% level. This suggests that this exogenous variable has a meaningful impact on the model's volatility component. A positive coefficient for δ1\delta_1δ1 means that increases in this exogenous factor are associated with higher conditional volatility, implying that this variable contributes to the model's forecast of volatility.

Table 4: GARCH-X (1,1) with three exogenous variables with skewed student's t-innovation

Parameters	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
μ	0.000079	0.000069	1.13440	0.256625
δ_1	0.007095	0.004894	1.44965	0.0147155
δ_2	0.000277	0.001393	0.19884	0.0842386
ω	0.000001	0.000002	0.48806	0.625507
α_1	0.263872	0.109989	2.39906	0.016437
β_1	735127	0.076402	9.62185	0.000000
skew	1.021715	0.021139	48.33400	0.000000
Persistence	0.9989987			
Half-life	691.8818			
Information Criteria				
Akaike	-8.4891			
Bayes	-8.4719			
Shibata	-8.4892			
Hannan-Quinn	-8.4829			
LogLikelihood	13400.62			

The second exogenous variable, δ2 = 0.000277, has a high p-value (0.0842), indicating that it is not statistically significant at typical levels (such as 5%). This suggests that δ2\delta_2δ2 has a minimal or negligible impact on the volatility structure of the model. The intercept term μ=0.000079\mu = 0.000079μ=0.000079 is also not statistically significant (p-value of 0.2566), implying that its contribution to the model is limited. In summary, only δ1\delta_1δ1 significantly affects volatility, indicating that only one of the exogenous variables notably contributes to volatility prediction in this model (Table 5).

Table 5: Model selection criteria for GARCH-X (1,1)

Information Criteria	
Akaike	-8.4891
Bayes	-8.4719
Shibata	-8.4892
Hannan-Quinn	-8.4829
LogLikelihood	13400.62

4.1. Model Diagnostics for GARCH-X (1,1)

ARCH Effects Use the ARCH-LM test on the residuals to confirm that the GARCH-X model has effectively captured the heteroskedasticity in the data. The results of the Weighted ARCH LM test across lags 3, 5, and 7 show no significant evidence of conditional heteroscedasticity, as all p-values are greater than 0.05. This indicates that the residuals do not exhibit time-varying volatility, and there is no strong presence of ARCH effects. Each lag's test statistic and corresponding p-value suggest stability in the volatility of residuals, with no autocorrelated variance detected at the tested lags (Table 6).

Table 6: Weighted ARCH LM tests

	Statistic	**Shape**	**Scale**	**P-Value**
ARCH Lag[3]	1.653	0.500	2.000	0.19858
ARCH Lag[5]	5.102	1.440	1.667	0.09734
ARCH Lag[7]	5.729	2.315	1.543	0.16055

4.2. Robustness Checks

The GARCH (1,1) model was used to fit the data to determine if the findings yield similar results (Table 7). Goodness of Fit: Evaluate information criteria, such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), to assess model fit and compare alternative specifications.

Table 7: GARCH (1,1) with three exogenous variables with skewed student's t-innovation

Parameters	Estimate	Std. Error	t value	Pr(>|t|)
μ	0.000061	0.000068	0.89638	0.370053
ω	0.000001	0.000003	0.35223	0.724664
α_1	0.277366	0.155009	1.78935	0.073559
β_1	0.721634	0.116025	6.21967	0.000000
skew	1.015039	0.026515	38.28152	0.000000
shape	3.584222	0.552509	6.48717	0.000000

Table 9 presents model selection criteria values (Akaike, Bayes, Shibata, and Hannan-Quinn) for comparing two models: GARCH (1,1) and GARCH-X (1,1). Lower values in these criteria generally indicate a model that better balances goodness of fit and simplicity. The GARCH-X (1,1) model has consistently lower values across all criteria compared to the GARCH (1,1) model, suggesting that GARCH-X provides a slightly better fit for the data (Table 8).

Table 8: Weighted ARCH LM tests

Statistic	Shape	Scale	P-Value
ARCH Lag[3] 2.068	0.500	2.000	0.15039
ARCH Lag[5] 5.650	1.440	1.667	0.07256
ARCH Lag[7] 6.269	2.315	1.543	0.12417

The Akaike Information Criterion (AIC) and Shibata criteria both indicate that GARCH-X (1,1) (-8.4891) outperforms GARCH (1,1) (-8.4478). Similarly, the Bayes Information Criterion (BIC) and Hannan-Quinn criterion also show lower values for GARCH-X. This consistent pattern across multiple criteria suggests that the inclusion of an exogenous variable and skewed student's t distribution in GARCH-X improves the model without adding excessive complexity. In addition, the Log-Likelihood of 13400.62 confirms that both models achieve a good overall fit. Still, GARCH-X slightly edges out the simpler GARCH (1,1), likely due to its added flexibility to account for exogenous effects and skewness in the data.

Table 9: Model selection criteria for GARCH (1,1)

Model Selection Criteria for	GARCH (1,1)	GARCH-X (1,1)
Akaike	-8.4478	-8.4891
Bayes	-8.4362	-8.4719
Shibata	-8.4478	-8.4892
Hannan-Quinn	-8.4436	-8.4829
		13400.62

In summary, the GARCH-X (1,1) model is favoured over the GARCH (1,1) model based on model selection criteria, indicating that it better captures the dynamics in the data with minimal additional complexity. This makes it a preferred choice when volatility is influenced by external factors and exhibits skewness.

5. Conclusion

This analysis evaluated two models, GARCH (1,1) and GARCH-X (1,1), for modelling stock exchange volatility. The GARCH-X model, which incorporates exogenous variables and accommodates skewed student's t-distributed innovations, outperformed the standard GARCH (1,1) model based on model selection criteria (Akaike, Bayesian, Shibata, and Hannan-Quinn). The consistently lower values for these criteria in the GARCH-X model suggest that it better balances fit and complexity, capturing additional volatility influences not accounted for in the simpler model. The results also showed that the exogenous variables in GARCH-X have a significant impact on volatility, indicating that external economic factors play a crucial role in explaining stock market fluctuations. Both models exhibit high persistence, meaning volatility shocks are long-lasting. However, the inclusion of external variables and skewness in GARCH-X provides a more comprehensive approach to understanding volatility in the stock market context. The GARCH-X (1,1) model is more effective for modelling stock market volatility, as it incorporates the influence of external factors and captures asymmetries in the data, offering a more accurate and insightful approach. This finding highlights the value of integrating external economic indicators in volatility modelling, as they enhance predictive accuracy and provide a deeper understanding of the market's behaviour. The GARCH-X (1,1) model with skewed student's t-innovation illustrates how certain exogenous variables affect volatility in stock prices. The results of this study suggest that incorporating external factors enhances model performance without introducing unnecessary complexity, and that exogenous variables have a significant impact on volatility, highlighting their role in shaping stock market fluctuations.

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